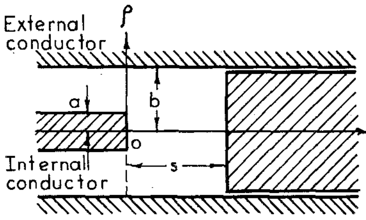

Fig. 1—Cutoff variable reactor.

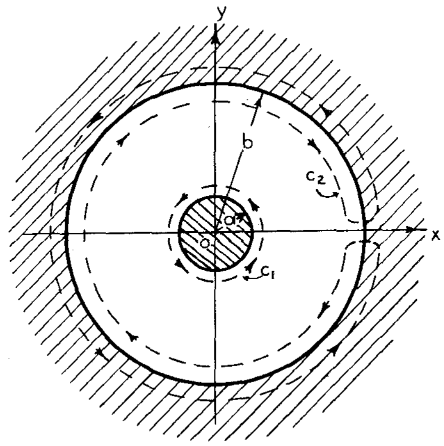

Fig. 2—Integral contour to obtain the input currents.

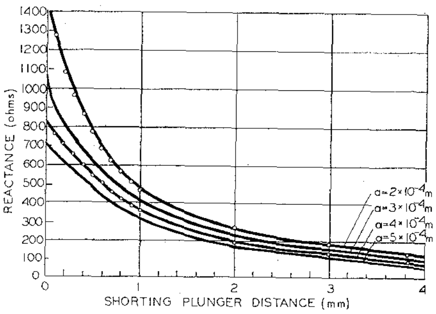

Fig. 3—Reactance of cutoff variable reactor.

where both A and B are integrating constants and both I_1 and K_1 are modified Bessel functions.

$$\sigma \equiv \sqrt{\left(\frac{\pi}{4s}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2} \tag{4}$$

and

$$B = \frac{bI_1(\sigma b) - aI_1(\sigma a)}{aK_1(\sigma a) - bK_1(\sigma b)} \cdot \tag{5}$$

The input voltage is given by

$$V = \int_a^b \left[-\frac{1}{j\omega\epsilon} \frac{\partial H_\phi}{\partial z} \right] d\rho. \tag{6}$$

The input reactance is given by dividing (6) by (2) providing (3),

$$X_S = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\frac{16\pi}{\lambda} ab\sigma s} \cdot \frac{\{I_0(\sigma b) - I_0(\sigma a)\}\{aK_1(\sigma a) - bK_1(\sigma b)\} - \{K_0(\sigma b) - K_0(\sigma a)\}\{bI_1(\sigma b) - aI_1(\sigma a)\}}{I_1(\sigma b)K_1(\sigma a) - I_1(\sigma_a)K_1(\sigma b)} \cdot \tag{7}$$

For example with $b = 2.25$ mm and $\lambda = 3.07$ cm, the reactance X_s changed as shown in Fig. 3 for various center conductor radii a and shorting plunger distances s. This reactor shows high reactance for small values of shorting plunger distances. When the inner conductor radius is 0.4 mm, in order to have 350 ohms, the conventional coaxial plunger requires a shorting plunger distance of 6.72 mm. On the other hand, the same reactance can be obtained by the cutoff reactor with a shorting plunger distance of 1 mm. This space economization is more effective for higher reactances. For 835 ohms, the shorting plunger distance of the conventional coaxial shorting plunger is 7.25 mm. For the same reactance, the shorting plunger distance of the cutoff reactor is 0.045 mm. The latter case was tested by experiment in a reflex klystron amplifier[1,2] circuit. The results of the experiment agreed with the calculations.

The author extends his thanks to R. Robinson, for numerical computation by IBM 650, and S. Krupnik and J. Stefancin for preparation of the manuscript.

KORYU ISHII
Dept. of Elec. Engrg.
Marquette University
Milwaukee, Wis.

[1] K. Ishii, "X-band receiving amplifier," *Electronics*, vol. 28, p. 202; April, 1955.

[2] K. Ishii, "Amplification circuit for the internal cavity-type reflex klystron," *J. Res. Inst. Tech., Nihon University* (Tokyo, Japan), vol. 14, pp. 1–10; December, 1957.

Single-Mode Cavity Maser at 2200 Mc*

Cavity masers which have been reported in the literature have utilized a dual-mode cavity resonant at the pump and signal frequencies, but Strandberg, *et al.*,[1] has reported an X-band cavity maser which can be operated with the cavity resonant only at the signal frequency. Siegmann[2] has also reported a multiplicity of resonances in his cavity maser for the pump signal which might possibly be caused by some sort of dielectric loading effect.

Both these facts suggested to us that an S-band cavity maser using only a single resonant mode at the signal frequency could be developed by using the ruby itself as the pump circuit. If this scheme were practical, design of tunable cavity masers at S-band would be much simpler than it is. With this thought in mind, a maser cavity circuit suitable for operation from 2100–2500 Mc was designed.

The maser's active material was pink ruby, $Al_2O_3Cr^{+++}$ oriented at an angle $\theta \cong 90°$. The cavity is a re-entrant $\lambda/4$-TEM stripline type in X-band waveguide with loop coupling as depicted in Fig. 1. The resonant frequency of the cavity is determined primarily by the length of center strip and by the ruby volume. The degree of coupling by the loop remained essentially constant over a 400-Mc frequency range. For the cavity filled on both sides as shown in Fig. 1, we estimated the filling factor to be approximately 75 per cent or greater. X-band pump energy is fed directly down the waveguide to the ruby crystals.

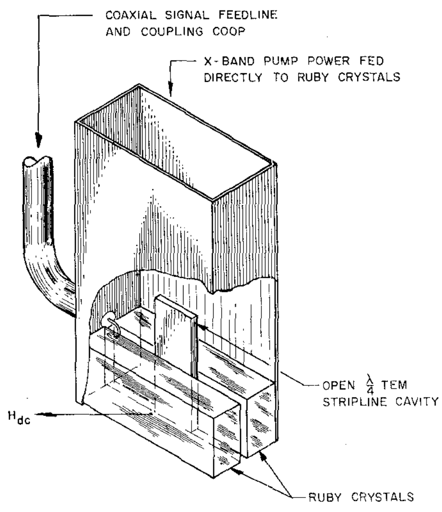

Fig. 1—$\lambda/4$ stripline cavity in standard X-band waveguide.

This cavity maser has been operated from 2120 to 2500 Mc by adjusting the length of the center strip and varying the pump frequency and dc magnetic field. Complete inversion of energy levels was obtained for all frequencies without the use of a pump cavity mode, and no pump resonances of any type were observed.

For the master amplifier characteristics considered here, the resonant frequency was 2200 Mc and the pump frequency was 12,470 Mc. Fig. 2 relates the gain-bandwidth products to the level of input power. This

* Received by the PGMTT, September 20, 1960.

[1] R. J. Morris, R. L. Kuhl, and M. W. P. Strandberg, "A tunable maser amplifier with large bandwidth," PROC. IRE, vol. 47, pp. 80–81; January, 1959.

[2] W. S. C. Chang, J. Cromack, and A. E. Siegman, "Experiments on a High-Performance Cavity Maser Using Ruby at S Band," Electron Devices Lab., Stanford Electronics Lab., Stanford University, Stanford, Calif., Rept. No. T.F. 156-4; July 21, 1959.

gain-bandwidth product curve is less than the calculated values, but there was no way to adjust and optimize the coupling once the loop was formed. In any tunable maser, some method of adjusting the coupling could be easily incorporated to offset this trouble. Fig. 3 shows pump power necessary for saturation vs signal input power. The amount of pump power necessary for saturation is larger than that usually required for the resonant cavity case, but not significantly so. We also noted that the gain of the maser is not dependent upon the frequency stability of the pump source. It was also noted that oscillations could be started over nearly the entire linewidth of the ruby, *i.e.*, 50-Mc tuning range of the pump frequency with little variation in the pump power.

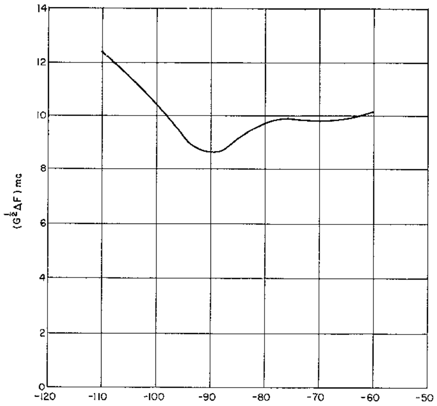

Fig. 2—Gain bandwidth product vs input level in dbm.

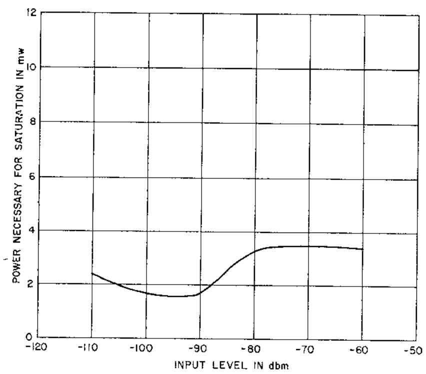

Fig. 3—Saturation pump power vs signal input level in dbm.

In conclusion, it appears feasible that a tunable cavity maser at S band could be developed without recourse to a dual-mode cavity system. This would greatly enhance the practicability of this type of amplifier for the 2000 Mc region where masers can be used efficiently for telemetry purposes.

The authors wish to thank Drs. P. S. Carter, Jr., and J. R. Singer for their help and criticisms.

D. W. Hanson
J. J. Rowley
Missile Systems Div.
Lockheed Aircraft Corp.
Sunnyvale, Calif.

Higher-Order Evaluation of Dipole Moments of a Small Circular Disk for Arbitrary Incident Fields*

In a recent note[1] the induced electric and magnetic dipole moments P and M due to the diffraction of a plane wave on a small circular disk were given. The expression for the electric dipole moment holds, however, only if the electric field vector is parallel to the plane of incidence. In a more general approach the case for an arbitrary primary field has been examined, and the following expressions have been obtained:

$$P_x = \frac{16}{3} a^3\epsilon_0 \left[E_x^i + \frac{(ka)^2}{30}\left(13E_x^i - \frac{3}{k^2}\frac{\partial^2 E_x^i}{\partial z^2} + \frac{2j}{\omega\epsilon_0}\frac{\partial H_z^i}{\partial y}\right) - j\frac{8}{9\pi}(ka)^3 E_x^i + 0((ka)^4)\right],$$

$$P_y = \frac{16}{3} a^3\epsilon_0 \left[E_y^i + \frac{(ka)^2}{30}\left(13E_y^i - \frac{3}{k^2}\frac{\partial^2 E_y^i}{\partial z^2} - \frac{2j}{\omega\epsilon_0}\frac{\partial H_z^i}{\partial x}\right) - j\frac{8}{9\pi}(ka)^3 E_y^i + 0((ka)^4)\right],$$

$$M_z = -\frac{8}{3} a^3 \left[H_z^i - \frac{(ka)^2}{10}\left(3H_z^i + \frac{1}{k^2}\frac{\partial^2 H_z}{\partial z^2}\right) + j\frac{4}{9\pi}(ka)^3 H_z^i + 0((ka)^4)\right].$$

The axis of the disk is along the z direction. The incident fields are evaluated at the center of the disk.

We now consider a plane wave ($\boldsymbol{E}^i$, $\boldsymbol{H}^i$) incident in the xz plane and with an angle of incidence θ. Using the expressions above we obtain

$$P_x = \frac{16}{3} a^3 \left[1 + \left(\frac{8}{15} - \frac{1}{10}\sin^2\theta\right)(ka)^2 - j\frac{8}{9\pi}(ka)^3 + \cdots \right] E_x^i,$$

$$P_y = \frac{16}{3} a^3 \left[1 + \left(\frac{8}{15} - \frac{1}{6}\sin^2\theta\right)(ka)^2 - j\frac{8}{9\pi}(ka)^3 + \cdots \right] E_y^i,$$

$$M_z = -\frac{8}{3} a^3 \left[1 - \frac{1}{10}(2 + \sin^2\theta)(ka)^2 + j\frac{4}{9\pi}(ka)^3 + \cdots \right] H_z^i.$$

P_x and M_z agree with the values given in reference [1].

W. H. Eggimann
Dept. Elec. Engrg.
Case Inst. Tech.
Cleveland, Ohio

* Received by the PGMTT, September 22, 1960. This work has been sponsored by the Electronics Res. Directorate of the AF Cambridge Res. Center under contract number AF 19(604)3887.

[1] W. H. Eggimann, "Higher order evaluation of dipole moments of a small circular disk," IRE Trans. on Microwave Theory and Techniques, vol. 8, p. 573; September, 1960.

Capacitance Definitions for Parametric Operation*

The purpose of this note is to reconcile the various definitions of nonlinear or equivalent time-varying capacitance which appear in the literature concerned with parametric devices. The problem considered is one of a nonlinear reactive element, let us say a capacitance, which is pumped by strong pump source at frequency f_p and which couples two circuit modes at frequencies f_s and f_i, usually called the signal and idling frequencies. For parametric operation we demand either $f_p = f_i + f_s$ or $f_p = f_i - f_s$. The nonlinear element (*i.e.*, a capacitance) has a charge-voltage characteristic given by

$$q = f(V). \tag{1}$$

Let us imagine that now we have applied a strong pump voltage V_p together with a dc bias voltage V_0 and subsequently we shall be concerned with the behavior upon application of small signal voltage δv at signal and idling frequencies. We can now define several capacitances:

1) The total capacitance C_T is defined as the ratio of total charge to total voltage, or

$$q = C_T V. \tag{2}$$

Obviously from (1)

$$C_T = \frac{f(V)}{V}; \tag{3}$$

this is the definition of capacitance used by Heffner and Wade.[1]

2) The incremental capacitance C_i is defined by

$$\delta q = C_i \delta V. \tag{4}$$

This is the definition used by Rowe.[2]

Two questions arise: first, in definition 1, what is the relationship between the time-varying capacitance produced by the pump alone to that which is effective in producing parametric action; second, what is the relationship between the two definitions of capacitance?

The first question already has been answered,[1] but it is perhaps worthwhile to add a few more details. If the biased capacitance is acted on by the pump voltage alone, then we would measure a charge,

$$q = (C_{T0} + C_{TP})(V_0 + V_p), \tag{5}$$

where we consider the capacitance to have a dc part C_{T0} and a time varying part C_{TP} varying sinusoidally at the pump frequency (for simplicity we will neglect harmonic terms). The time varying part could be measured or could be inferred from a static plot of the nonlinear characteristic. We necessarily demand that the amplitude of the time-varying part be less than or at most equal to the dc part in order to have the

* Received by the PGMTT, October 4, 1960.

[1] H. Heffner and G. Wade, *J. Appl. Phys.*, vol. 29, pp. 1321–1331; September, 1958.

[2] H. E. Rowe, "Some general properties of nonlinear elements—II. Small signal theory," Proc. IRE, vol. 46, pp. 850–860; May, 1958.